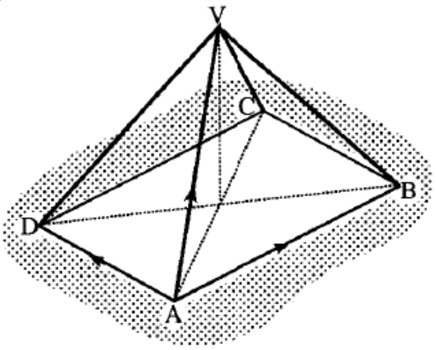
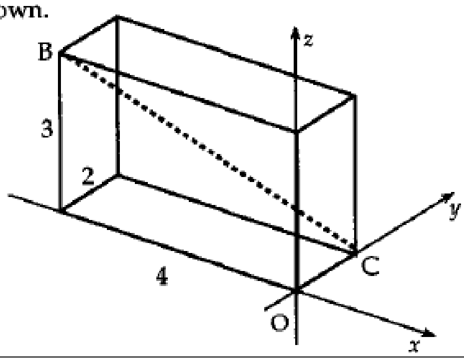
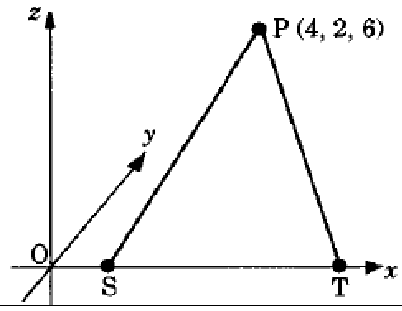
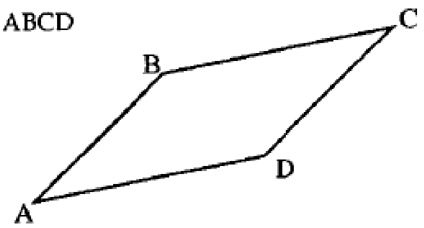
Vectors

Components, Magnitude and Unit Vectors

2006 P2	**6.** P is the point (−1, 2, −1) and Q is (3, 2, −4). (*a*) Write down $\overrightarrow{PQ}$ in component form. (*b*) Calculate the length of $\overrightarrow{PQ}$. (*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$.	 **1** **1** **1**
2016 P1	**7.** Three vectors can be expressed as follows: $\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$ $\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (a) Find $\overrightarrow{FH}$. (b) Hence, or otherwise, find $\overrightarrow{FE}$.	 **2** **2**

Pre 2000 – Components, Magnitude and Unit Vectors

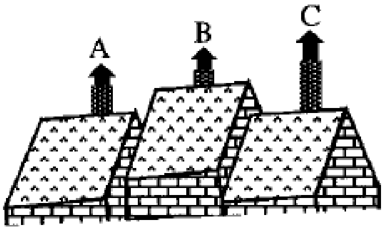
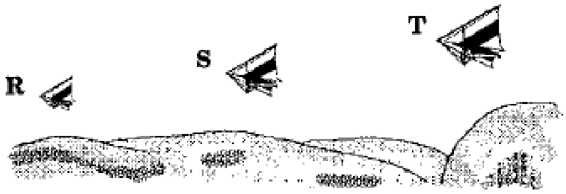
1	VABCD is a pyramid with rectangular base ABCD. The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by $\overrightarrow{AB} = 8i + 2j + 2k$ $\overrightarrow{AD} = -2i + 10j - 2k$ and $\overrightarrow{AV} = i + 7j + 7k$. Express $\overrightarrow{CV}$ in component form.  	**3**
2	The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively. (*a*) Express $\overrightarrow{PQ}$ in component form. (*b*) Find the length of PQ.	 2 1
3	Calculate the length of the vector $2\mathbf{i} - 3\mathbf{j} + \sqrt{3}\mathbf{k}$.	2
4	A is the point (−3,2,4) and B is (−1,3,2). Find (*a*) the components of vector $\overrightarrow{AB}$; (*b*) the length of AB.	 1 2

5	A cuboid crystal is placed relative to the coordinate axes as shown. (*a*) Write down $\overrightarrow{BC}$ in component form. (*b*) Calculate $\lvert\overrightarrow{BC}\rvert$. 	2
6	The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows: $\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$. (*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ *and* $\boldsymbol{k}$. (*b*) Find the value of $\lvert 2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}\rvert$.	1 2
7	The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 	3
8	Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by $\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k}$ and $\boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}$. (*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. (*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ (*c*) Find $\lvert\boldsymbol{r}\rvert$.	2 1 1
9	A is the point (2, –1 ,4), B is (7, 1, 3) and C is (–6, 4, 2). If ABCD is a parallelogram, find the coordinates of D. 	3
10	PQRS is a parallelogram with vertices P(1, 3, 3), Q(4, –2, –2) and R(3, 1, 1). Find the coordinates of S.	3

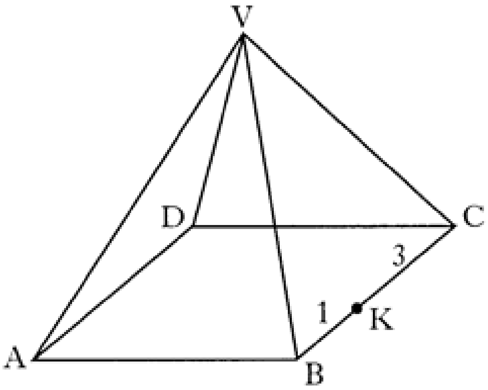
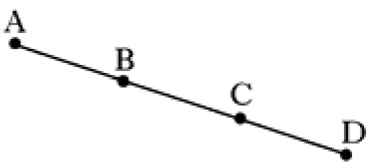
Collinearity

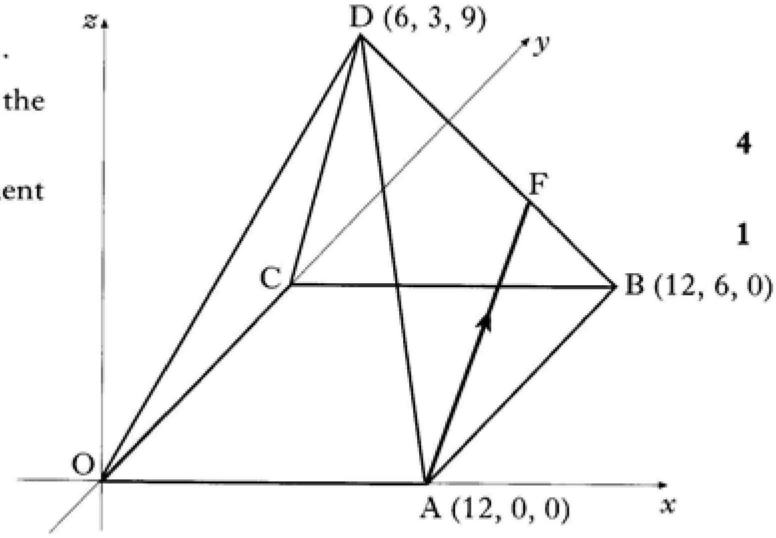
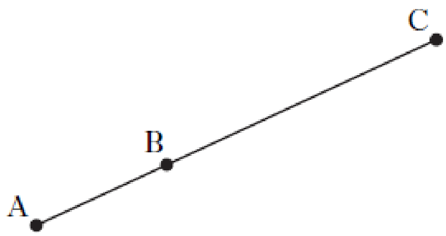
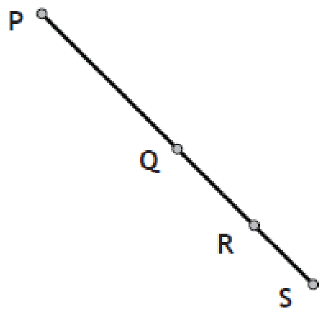
8.(JAN) 02 P2	With reference to a suitable set of coordinate axes, A, B and C are the points (–8, 10, 20), (–2, 1, 8) and (0, –2, 4) respectively. Show that A, B and C are collinear and find the ratio AB : BC.	**4**

Pre 2000 - Collinearity

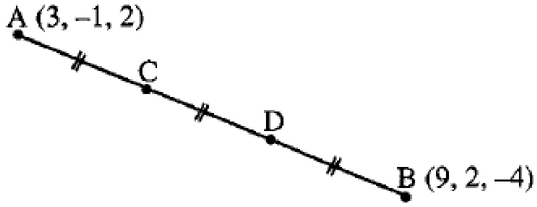
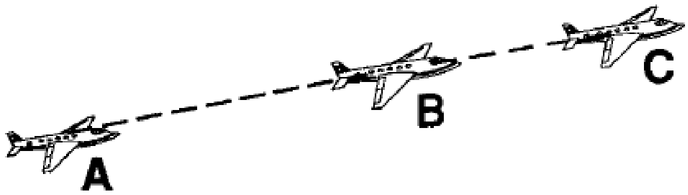
1	Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1, 4) and C(4, –9, 8). Show that A, B and C are collinear. 	3
2	A is the point (2, –5, 6), B is (6, –3, 4) and C is (12, 0, 1). Show that A, B and C are collinear and determine the ratio in which B divides AC.	4
3	Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6). Prove that R, S and T are collinear. 	3
4	(a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. (b) Find the ration in which M divides LN.	4 1
5	Show that P(2, 2, 3), Q(4, 4, 1) and R(5, 5, 0) are collinear and find the ratio in which Q divides PR.	4

Section Formula

2000 P1	**B7.** VABCD is a pyramid with a rectangular base ABCD. Relative to some appropriate axes, $\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$ $\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$ $\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$. K divides BC in the ratio 1:3. Find $\overrightarrow{VK}$ in component form. 	3
2002 P1	**2.** The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1. Find the coordinates of Q.	3
2003 P1	**6.** A and B are the points (–1, –3, 2) and (2, –1, 1) respectively. B and C are the points of trisection of AD, that is AB = BC = CD. Find the coordinates of D. 	3

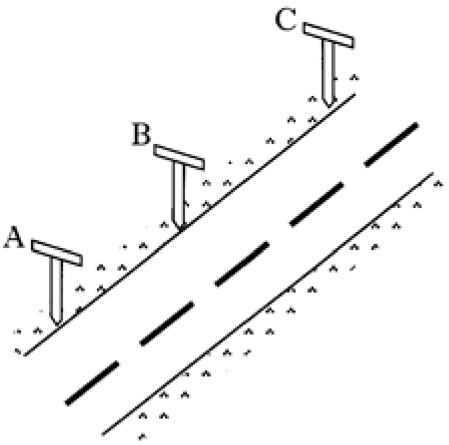
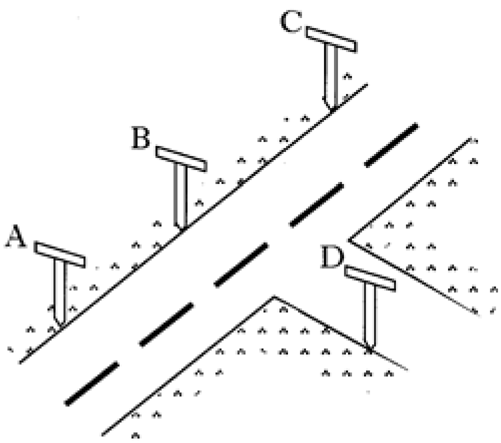
2004 P1	**5.** A, B and C have coordinates (–3, 4, 7), (–1, 8, 3) and (0, 10, 1) respectively. (*a*) Show that A, B and C are collinear. **3** (*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$. **2**
2005 P1	**3.** D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9). F divides DB in the ratio 2:1. (*a*) Find the coordinates of the point F. **4** (*b*) Express $\overrightarrow{AF}$ in component form. **1** 
2007 P1	**2.** Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively. A, B and C are collinear points and C is positioned such that BC = 2AB. Find the coordinates of C. **4** 
2015 SP P1	**3.** In the diagram, P has coordinates (–6, 3, 9), $\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$. Find the coordinates of S. **5** 
2016 P1	**11.** (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively. Point B divides AC in the ratio $1 : 2$. Find the coordinates of B. **2** (b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$. Determine the value of k. **3**

Pre 2000 - Section Formula

1	The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).  (*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (*b*) Find the coordinates of C and D.	 2 2
2	An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point (–1, 3, 4) and B is the point (3, 1, –2). Find the co-ordinates of the point C.  	3
3	ABCD is a quadrilateral with vertices A(4, –1, 3), B(8, 3, –1), C(0, 4, 4) and D(–4, 0, 8). (*a*) Find the coordinates of M, the midpoint of AB. (*b*) Find the coordinates of the point T, which divides CM in the ratio 2:1. (*c*) Show that B, T and D are collinear and find the ratio in which T divides BD.	 **(1)** **(3)** **(4)**

The Dot Product

2000 P2	**B7.** For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?	**2**

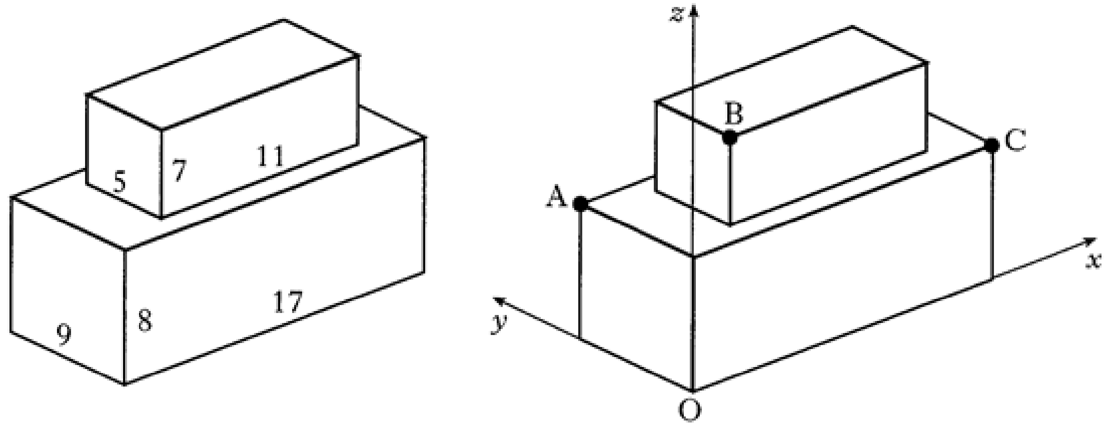
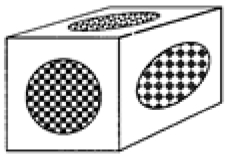
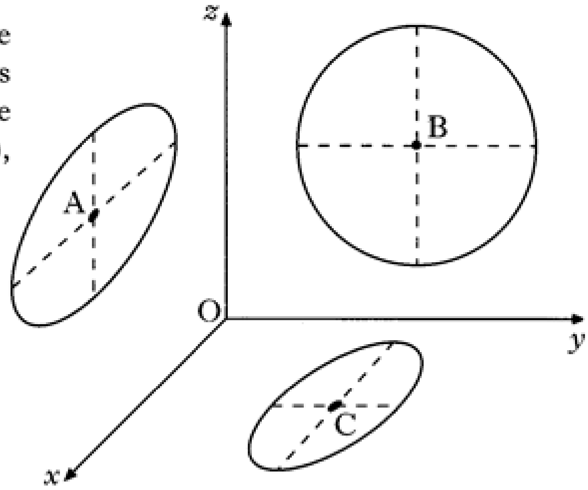
2001 P1	**3.** (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B(–2, –1, 1) and C(6, 11, 5). Determine whether or not the section of road ABC has been built in a straight line.  **3** (*b*) A further T-rod is placed such that D has coordinates (1, –4, 4). Show that DB is perpendicular to AB.  **3**
6.(JAN) 02 P1	(*a*) If $\boldsymbol{u} = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$. 2 (*b*) Hence, or otherwise, show that $\boldsymbol{u} + 3\boldsymbol{v}$ and $\boldsymbol{u} - 3\boldsymbol{v}$ are perpendicular. 2
2003 P1	**3.** Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$. Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**
2009 P1	**22.** D, E and F have coordinates (10, –8, –15), (1, –2, –3) and (–2, 0, 1) respectively. (*a*) (i) Show that D, E and F are collinear. (ii) Find the ratio in which E divides DF. **4** (*b*) G has coordinates (k, 1, 0). Given that DE is perpendicular to GE, find the value of k. **4**
2013 P1	**24.** (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear. (ii) Find the ratio in which T divides AB. **4** (*b*) The point C lies on the x-axis. If TB and TC are perpendicular, find the coordinates of C. **5**

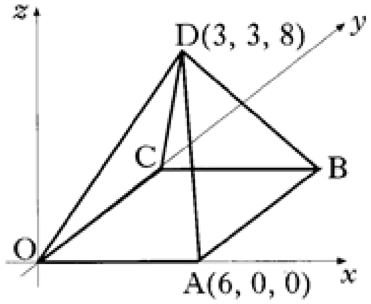
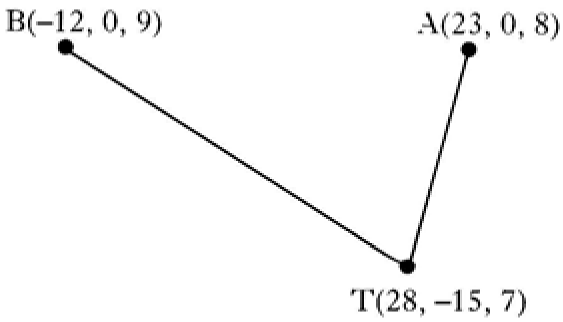
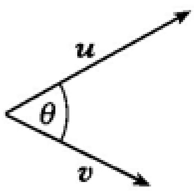
2015 SP P2	6. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that AB is perpendicular to CD. Determine the value of x. **5**
2015 P1	1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular. Determine the value of t. **2**

Pre 2000 – The Dot Product

1	Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by $\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k}$ and $\boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}$. (*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. (*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ (*c*) Find $\lvert \boldsymbol{r} \rvert$.	 2 1 1
2	If $\boldsymbol{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$. Hence show that $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular.	3
3	Show that the vectors $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ are perpendicular.	3
4	A(4, 4, 10), B(−2, −4, 12) and C(−8, 0, 10) are the vertices of a right-angled triangle. Determine which angle of the triangle is the right angle.	3
5	Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$ are perpendicular.	3
6	The vector $a\boldsymbol{i} + b\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to both the vectors $\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $-2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$. Find the values of a and b.	3
7	Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular. Determine the value of t.	2

Calculating Angles

2000 P2	**B9.** A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm. Coordinate axes are taken as shown.  (*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B. **1** (*b*) Calculate the size of angle ABC. **6**
2001 P2	**4.** A box in the shape of a cuboid is designed with **circles** of different sizes on each face.  The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0). Find the size of angle ABC. **7** 

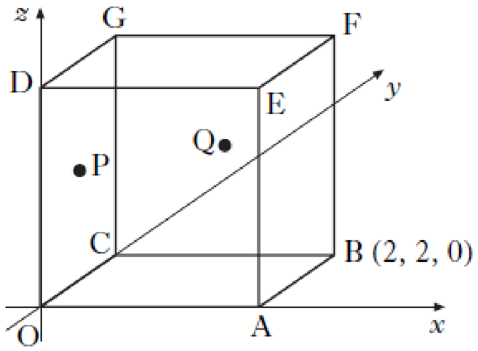
2002 P2	**2.** The diagram shows a square-based pyramid of height 8 units. Square OABC has a side length of 6 units. The coordinates of A and D are (6, 0, 0) and (3, 3, 8). C lies on the y-axis.  (*a*) Write down the coordinates of B. **1** (*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2** (*c*) Calculate the size of angle ADB. **4**
2004 P2	**2.** P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively. (*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. **2** (*b*) Hence or otherwise find the size of angle PQR. **5**
2005 P2	**4.** The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops. Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(–12, 0, 9) and T(28, –15, 7). In the dark, Andrew and Bob locate Tracy using heat-seeking beams.  (*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2** (*b*) Calculate the angle between these two beams. **5**
2006 P1	**9.** $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.  (*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2** (*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5** (*c*) Deduce the only possible value of k. **1** (*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

2007 P2

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.



(*a*) Write down the coordinates of G. **1**

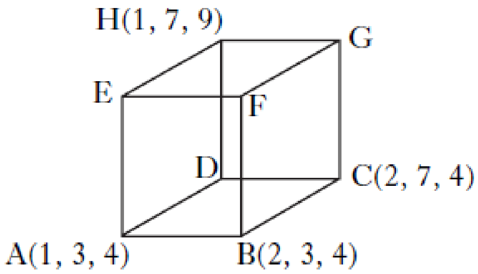
(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(*c*) Find the size of angle POQ. **5**

2008 SP1 P2

2. The diagram shows a wire framework in the shape of a cuboid with the edges parallel to the axes.

Relative to these axes, A, B, C and H have coordinates (1, 3, 4), (2, 3, 4), (2, 7, 4) and (1, 7, 9) respectively.



(*a*) State the lengths of AB, AD and AE. **1**

(*b*) Write down the components of $\overrightarrow{HB}$ and $\overrightarrow{HC}$ and hence or otherwise calculate the size of angle BHC. **7**

2008 SP2 P2

1. Given that $\overrightarrow{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$, find the size of angle PQR. **5**

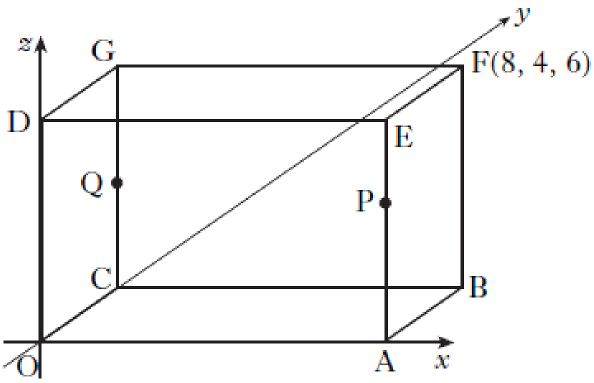
2008 P2

2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

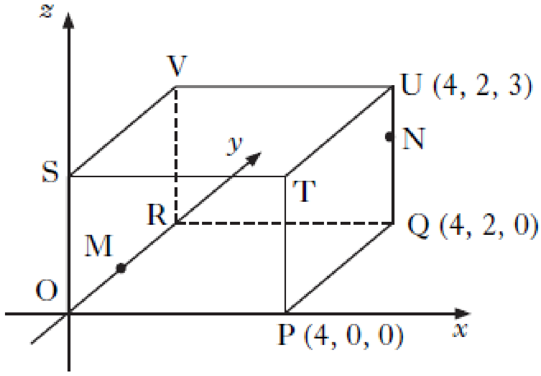
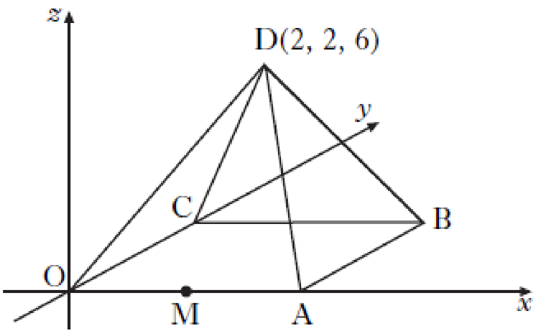
Q is the midpoint of CG.



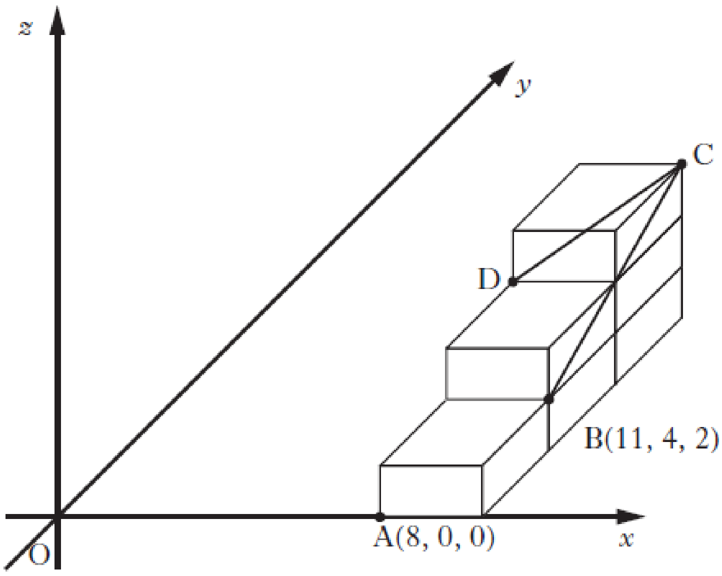
(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

(*c*) Find the size of angle QPA. **5**

2010 P2	**1.** The diagram shows a cuboid OPQR,STUV relative to the coordinate axes. P is the point (4, 0, 0), Q is (4, 2, 0) and U is (4, 2, 3). M is the midpoint of OR. N is the point on UQ such that $UN = \frac{1}{3}UQ$.  (*a*) State the coordinates of M and N. (*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. (*c*) Calculate the size of angle MVN.	 2 2 5
2011 P2	**1.** D,OABC is a square based pyramid as shown in the diagram below.  O is the origin, D is the point (2, 2, 6) and OA = 4 units. M is the mid-point of OA. (*a*) State the coordinates of B. (*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. (*c*) Find the size of angle BDM.	 1 3 5
2012 P2	**5.** A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, *k*, 0). (*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form. (ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2+6k+14)}}$. (*b*) If angle ABC = 30°, find the possible values of *k*.	 7 5

2014 P2	**4.** Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.

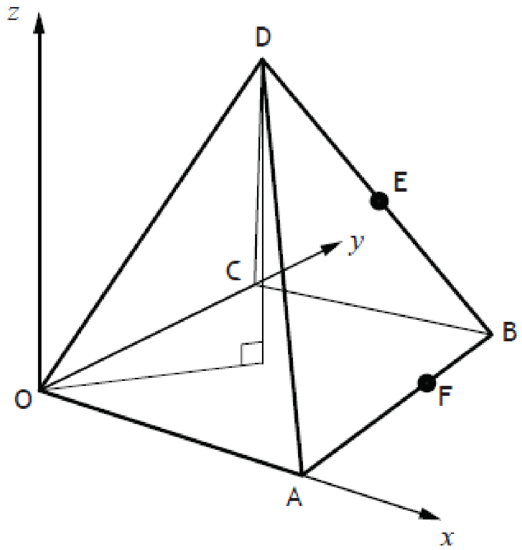


A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. **2**

(*c*) Find the size of the angle BCD. **5**

<table>
<tr><td>2015
SP P2</td><td>1.


A square based right pyramid is shown in the diagram.

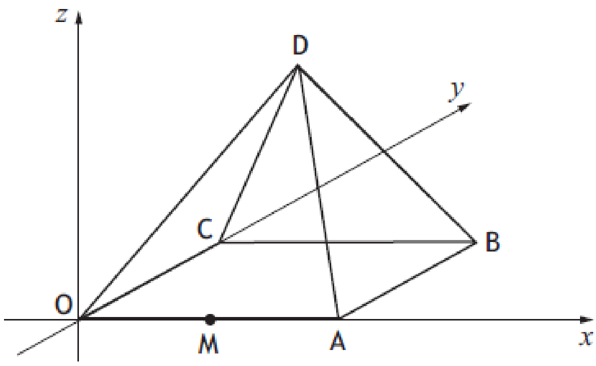
Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F. — 2

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. — 2

(c) Hence, or otherwise, calculate the size of angle DEF. — 4</td></tr>
<tr><td>2015
EP P2</td><td>5. D,OABC is a square-based pyramid as shown.


O is the origin and OA = 4 units.

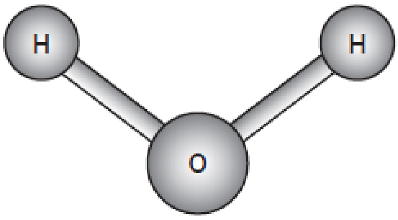
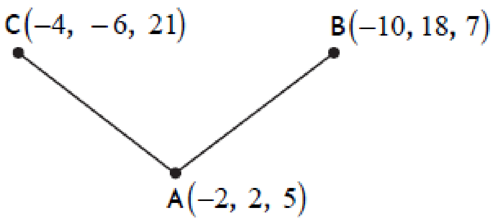
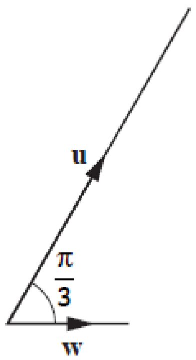
M is the mid-point of OA.

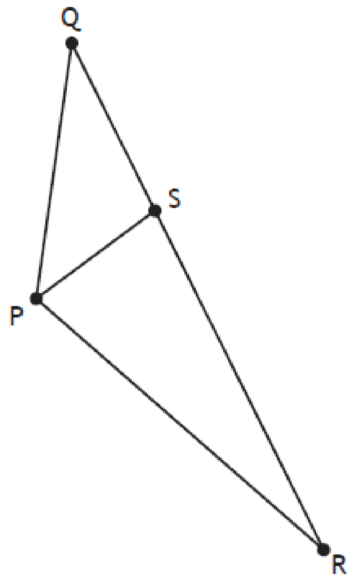
$\overline{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$. — 1

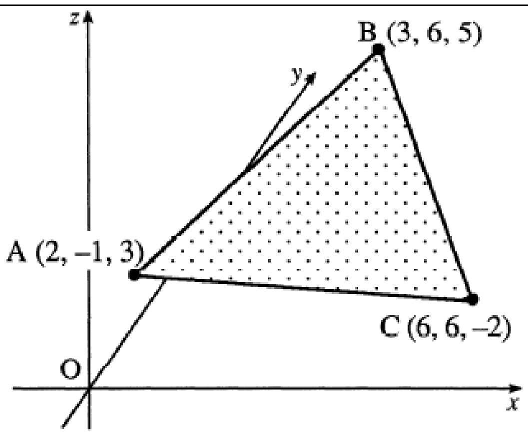
(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. — 3

(c) Find the size of angle BDM. — 5</td></tr>
</table>

2016 P2	**5.** The picture shows a model of a water molecule.  Relative to suitable coordinate axes, the oxygen atom is positioned at point A$(-2,\ 2,\ 5)$. The two hydrogen atoms are positioned at points B$(-10, 18, 7)$ and C$(-4,\ -6,\ 21)$ as shown in the diagram below.  (a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2** (b) Hence, or otherwise, find the size of angle BAC. **4**
2017 P1	**5.** Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively. (a) Evaluate $\mathbf{u}.\mathbf{v}$. **1** (b)  Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $\lvert\mathbf{w}\rvert = \sqrt{3}$. Calculate $\mathbf{u}.\mathbf{w}$. **3**

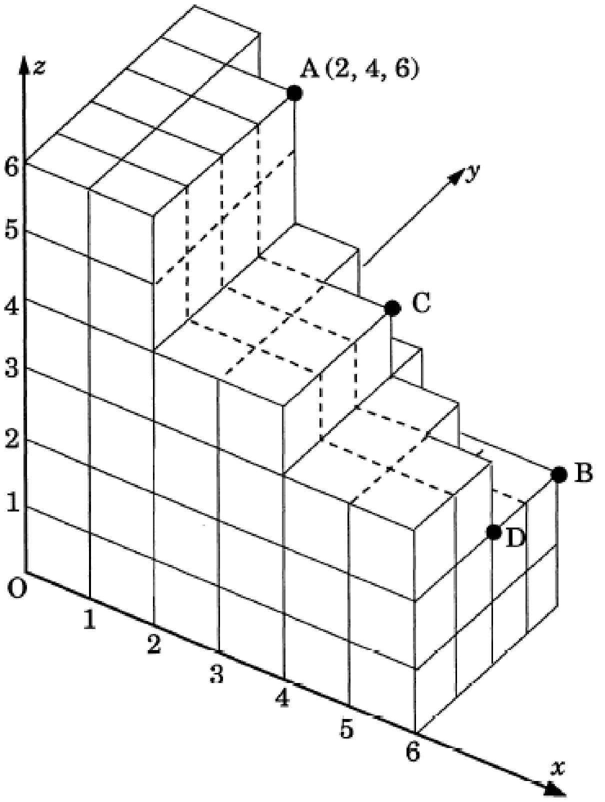
2017 P2	**5.** In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.  (a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **2** The point S divides QR in the ratio 1:2. (b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **2** (c) Hence, find the size of angle QPS. **5**

Pre 2000 - Calculating Angles

1	A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2). (*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (*b*) Calculate the size of angle BAC. (*c*) Hence find the area of the triangle.  	**(2)** **(5)** **(2)**

2

With coordinate axes as shown, the point A is (2,4,6).



(*a*) Write down the coordinates of B,C and D. **(3)**

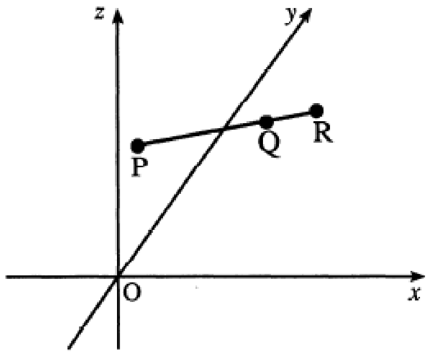
(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

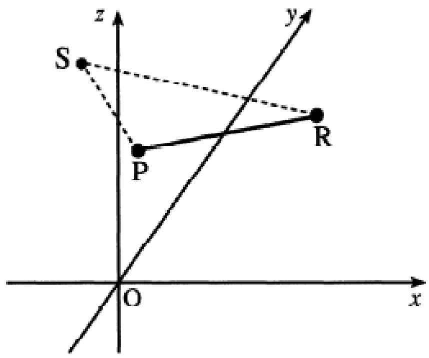
(*d*) Hence calculate the size of angle OAB. **(2)**

3

Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



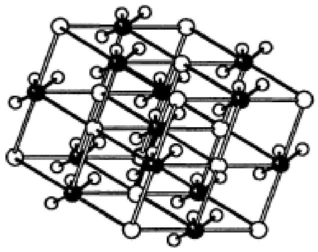
(*a*) Find the coordinates of R. **(3)**



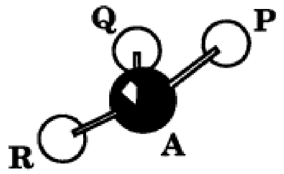
(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**

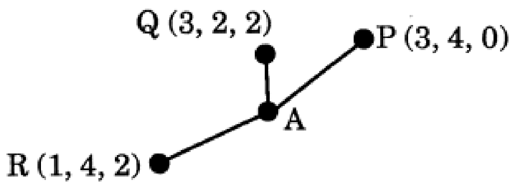
4

The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

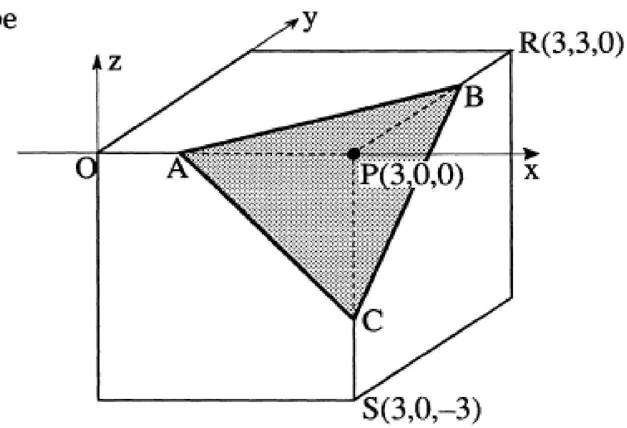
(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

5

A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC.
Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.

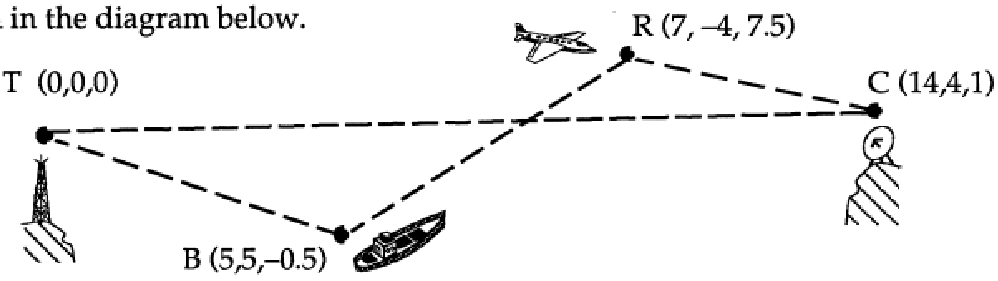


(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

6 Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

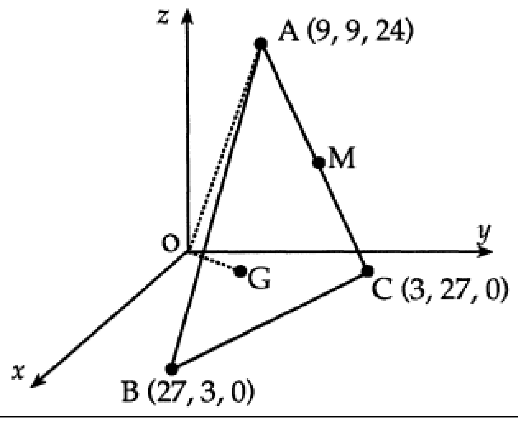
(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

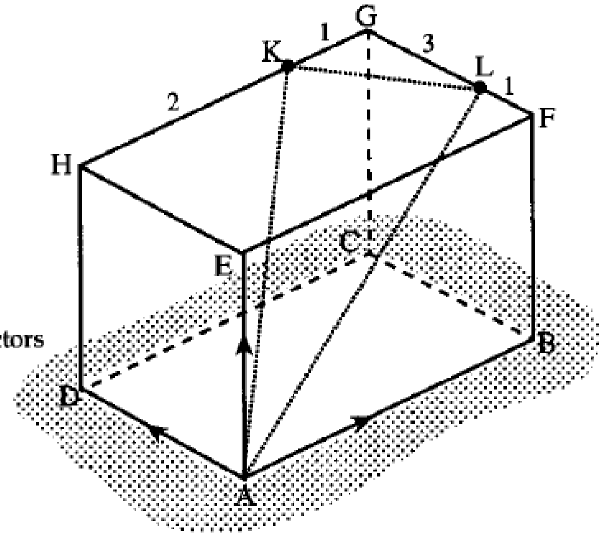
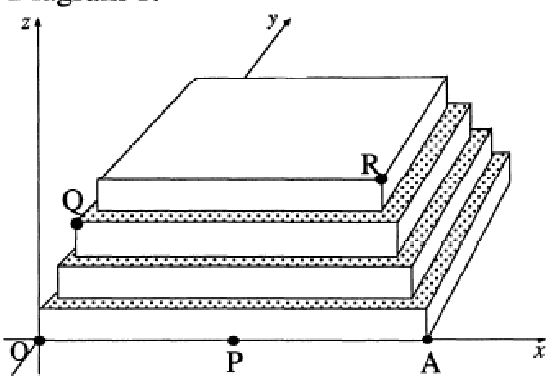
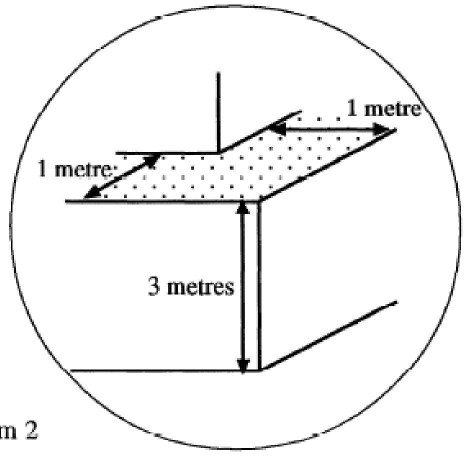
(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

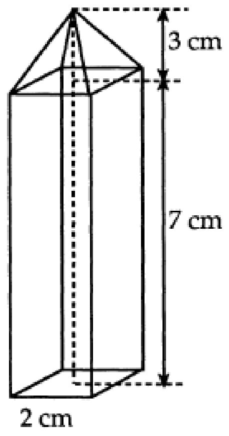
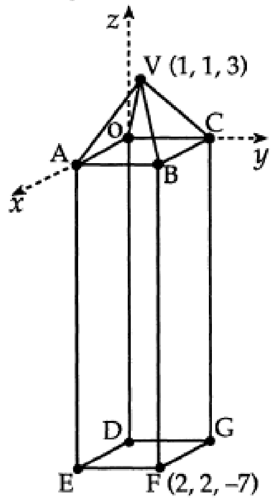
(*d*) Calculate the size of angle TCR. **(5)**

7 (*a*) Relative to mutually perpendicular axes O*x*, O*y* and O*z*, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.
Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

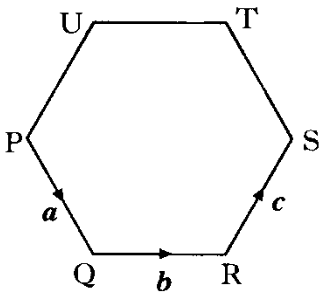
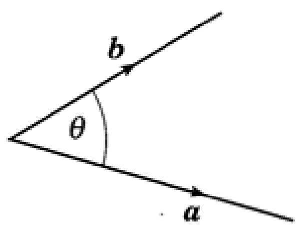
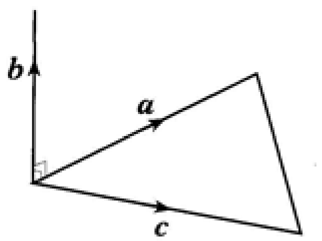
(*b*) Calculate the size of angle GOA. **(5)**

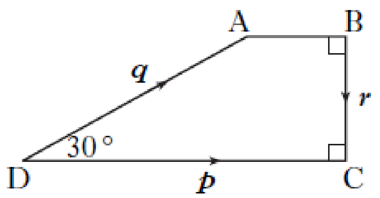
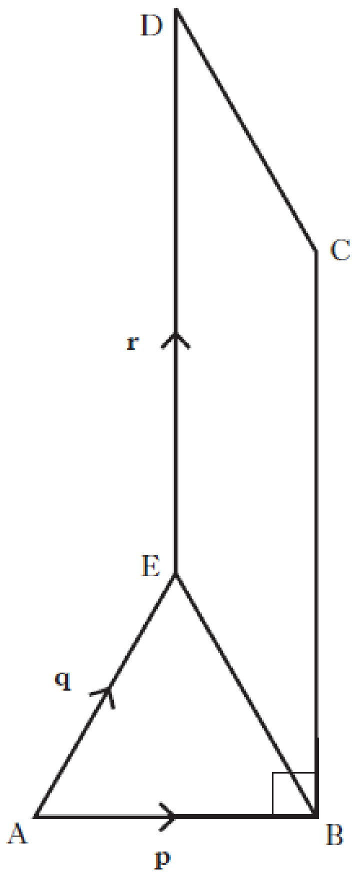
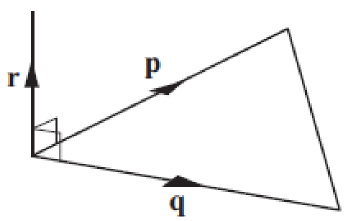


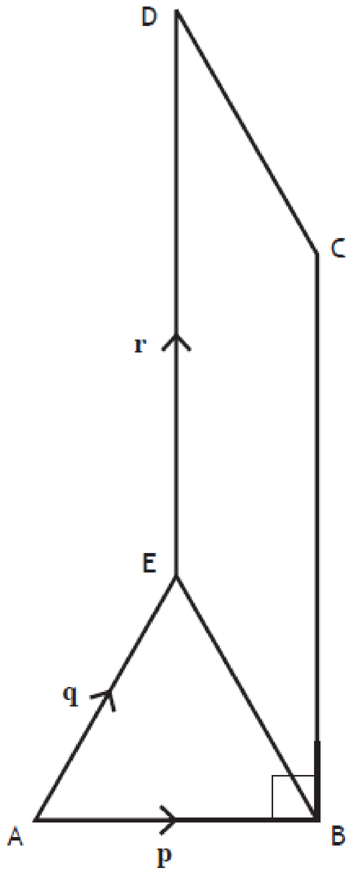
8	ABCDEFGH is a cuboid. K lies two thirds of the way along HG. (i.e. HK:KG = 2:1). L lies one quarter of the way along FG. (i.e. FL:LG = 1:3). $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ can be represented by the vectors $\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\begin{pmatrix}-8\\4\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-3\\5\end{pmatrix}$ respectively.  (*a*) Calculate the components of $\overrightarrow{AK}$. **2** (*b*) Calculate the components of $\overrightarrow{AL}$. **2** (c) Calculate the size of angle KAL. **5**
9	The first four levels of a stepped pyramid with a square base are shown in Diagram 1.  Diagram 1  Diagram 2 Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m. The height and "width" of a step at a corner are shown in the enlargement in Diagram 2. With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0). (*a*) Find the coordinates of Q and R. **(2)** (*b*) Find the size of angle QPR. **(7)**

10	Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ. The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$ and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$  Diagram 1  Diagram 2 (*a*) Find (i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$; (ii) the size of angle EVF. **(7)** (*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it. Calculate the area of the glass triangle VEF. **(3)**	

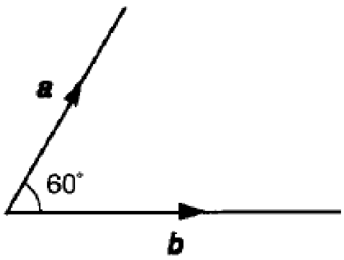
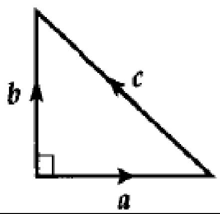
Algebraic Properties of the Dot Product

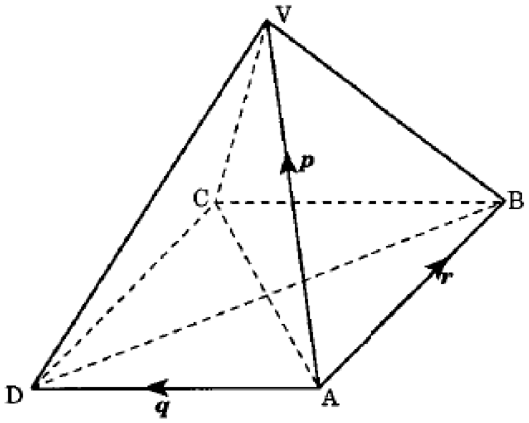
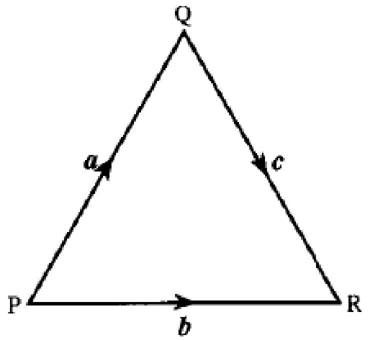
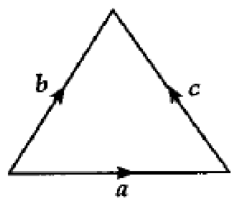
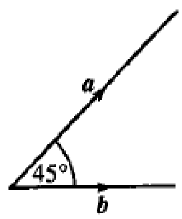
7.(JAN) 02 P1	PQRSTU is a regular hexagon of side 2 units. $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively. Find the value of $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.  	3
2003 P2	**9.** The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. If $\lvert\boldsymbol{a}\rvert = 5$, $\lvert\boldsymbol{b}\rvert = 4$ and $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$.  	4
2005 P2	**10.** Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$. Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$.  	4

2009 P2	7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°. It is also given that $\lvert\boldsymbol{p}\rvert = 4$ and $\lvert\boldsymbol{q}\rvert = 3$.  (a) Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})$. (b) Find $\lvert\boldsymbol{q}+\boldsymbol{r}\rvert$ and $\lvert\boldsymbol{p}-\boldsymbol{q}\rvert$.	 6 4
2015 OLD P2	6. Vectors **p**, **q** and **r** are represented on the diagram as shown. • BCDE is a parallelogram • ABE is an equilateral triangle • $\lvert\mathbf{p}\rvert = 3$ • Angle ABC = 90°  (a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$. (b) Express $\overrightarrow{EC}$ in terms of **p**, **q** and **r**. (c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $\lvert\mathbf{r}\rvert$.	 3 1 3
2015 EP P2	6. An equilateral triangle with sides of length 3 units is shown.  Vector **r** is 2 units long and is perpendicular to both vectors **p** and **q**. Calculate the value of the scalar product $\mathbf{p}.(\mathbf{p}+\mathbf{q}+\mathbf{r})$.	 4

2015 P2	**6.** Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown. • BCDE is a parallelogram • ABE is an equilateral triangle • $\lvert\mathbf{p}\rvert = 3$ • Angle ABC = 90°  (a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$. **3** (b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. **1** (c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \frac{9}{2}$, find $\lvert\mathbf{r}\rvert$. **3**

Pre 2000 - Algebraic Properties of the Dot Product

1	The diagram shows representatives of two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, inclined at an angle of 60°. If $\lvert\boldsymbol{a}\rvert = 2$ and $\lvert\boldsymbol{b}\rvert = 3$, evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b})$  	3
2	The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. The two equal sides have length 2 units. Find the value of $\boldsymbol{b}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$  	5

3	In the square-based pyramid, all the eight edges are of length 3 units. $\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$. Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$. 	4
4	PQR is an equilateral triangle of side 2 units. $\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$. Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. 	4
5	The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined as follows: $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{c} = -\boldsymbol{j} + \boldsymbol{k}$. (*a*) Evaluate $\boldsymbol{a}.\boldsymbol{b} + \boldsymbol{a}.\boldsymbol{c}$. (*b*) From your answer to part (*a*), make a deduction about the vector $\boldsymbol{b} + \boldsymbol{c}$.	3 2
6	The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown. Evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$. 	5
7	The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $\lvert \boldsymbol{a} \rvert = 3$ and $\lvert \boldsymbol{b} \rvert = 2\sqrt{2}$. These vectors are inclined at an angle of 45° to each other. (*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$ (ii) $\boldsymbol{b}.\boldsymbol{b}$ (iii) $\boldsymbol{a}.\boldsymbol{b}$ (*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$. Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $\lvert \boldsymbol{p} \rvert$. 	2 4